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Hyperons analogous to the $\Lambda(1405)$

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Abstract The low mass of the $\Lambda(1405)$ hyperon with $j^P = 1/2^-$, which is higher than the ground state $\Lambda(1116)$ mass by 290 MeV, is difficult to understand in quark models. We analyze the hyperon spectrum in the bound state approach of the Skyrme model that successfully describes both the $\Lambda(1116)$ and the $\Lambda(1405)$. This model predicts that several hyperon resonances of the same spin but with opposite parity form parity doublets that have a mass difference of around 300 MeV, which is indeed realized in the observed hyperon spectrum. Furthermore, the existence of the $\Xi(1620)$ and the $\Xi(1690)$ of $j^P = 1/2^-$ is predicted by this model. Comments on the Ω baryons and heavy quark baryons are made as well.

1 Introduction

The low mass of the $\Lambda(1405)$ hyperon has been a puzzle when it is described as a P -wave three-quark state [1,2]. Instead, interpreting the $\Lambda(1405)$ as a $\bar{K}N$ bound state has been successful to understand various properties of the $\Lambda(1405)$ [3,4,5]. It is then natural to search for other hyperons that have similar structure as the $\Lambda(1405)$. In this paper, we investigate hyperon spectrum in the bound state approach in the Skyrme model to search for the hyperons analogous to the $\Lambda(1405)$. We will also make a short comment on the heavy-flavored analog of the $\Lambda(1405)$.

In the bound state approach to the Skyrme model [6], hyperons are described as bound states of the soliton and mesons carrying strangeness quantum number such as the kaon and the K^* vector mesons. The underlying dynamics between the soliton and kaon is described by the Lagrangian of meson degrees of freedom. As shown in Ref. [6], the Wess-Zumino term in an $SU(3)$ chiral Lagrangian pushes up the $S = +1$ state and pulls down the $S = -1$ state. As a result, the $S = +1$ pentaquark Θ^+ cannot be a bound state, while the $S = -1$ states correspond to the normal hyperons. We refer to Ref. [7] for further discussions on the Θ^+ in this model. Furthermore, this model renders two kinds of bound state, one in P -wave and one in S -wave. The P -wave state is strongly bound and, when quantized, it gives the ground states of hyperons with $j^P = 1/2^+$ and $3/2^+$. On the other hand, the S -wave state is an excited state and, when quantized, it corresponds to the $\Lambda(1405)$ with $j^P = 1/2^-$. Therefore, this model gives a natural way to describe both the $\Lambda(1116, 1/2^+)$ and the $\Lambda(1405, 1/2^-)$ on the same ground [8].

Here, we present the results on the hyperon spectrum based on this approach. We found that there exists a pattern in the hyperon spectrum, which can be verified by the measured hyperon masses. In particular, the predicted Ξ and Ω spectra are very different from the quark model predictions and can explain several puzzles in hyperon spectrum, which can be further tested by experiments at current accelerator facilities.

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Table 1 Low-lying Ξ and Ω baryon spectrum of spin 1/2 and 3/2 predicted by the non-relativistic quark model of Chao et al. (CIK) [10], relativized quark model of Capstick and Isgur (CI) [11], Glozman-Riska model (GR) [12], large N_c analysis [13, 14, 15, 16, 17], algebraic model (BIL) [18], QCD sum rules (QCD-SR) [19], and the recent nonrelativistic quark model of Pervin and Roberts (PR) [20]. The mass is given in the unit of MeV.

State	CIK	CI	GR	Large- N_c	BIL	QCD-SR	PR
$\Xi(\frac{1}{2}^+)$	1325	1305	1320		1334	1320	1325
	1695	1840	1798	1825	1727		1891
	1950	2040	1947	1839	1932		2014
$\Xi(\frac{3}{2}^+)$	1530	1505	1516		1524		1520
	1930	2045	1886	1854	1878		1934
	1965	2065	1947	1859	1979		2020
$\Xi(\frac{1}{2}^-)$	1785	1755	1758	1780	1869	1550	1725
	1890	1810	1849	1922	1932		1811
	1925	1835	1889	1927	2076		
$\Xi(\frac{3}{2}^-)$	1800	1785	1758	1815	1828	1840	1759
	1910	1880	1849	1973	1869		1826
	1970	1895	1889	1980	1932		
$\Omega(\frac{1}{2}^+)$	2190	2220	2068	2408	2085		2175
	2210	2255	2166		2219		2191
$\Omega(\frac{3}{2}^+)$	1675	1635	1651		1670		1656
	2065	2165	2020	1922	1998		2170
	2215	2280	2068	2120	2219		2182
$\Omega(\frac{1}{2}^-)$	2020	1950	1991	2061	1989		1923
$\Omega(\frac{3}{2}^-)$	2020	2000	1991	2100	1989		1953

2 Hyperon Spectrum

There have been many models on the structure of hyperons, mostly based on quark dynamics. As shown in Table 1, the hyperon mass spectrum have been predicted by various models. It reveals that the predictions are highly model-dependent and all these quark models leave a puzzle on the low mass of the $\Xi(1620)$ and the $\Xi(1690)$, which is very similar to the puzzle of the $\Lambda(1405)$. In the bound state approach in the Skyrme model, the mass of a hyperon with isospin i and spin j is written as [9]

$$\begin{aligned}
M(i, j, j_m) = & M_{\text{sol}} + n_1 \omega_1 + n_2 \omega_2 \\
& + \frac{1}{2\mathcal{I}} \left\{ i(i+1) + c_1 c_2 j_m(j_m+1) + (\bar{c}_1 - c_1 c_2) j_1(j_1+1) + (\bar{c}_2 - c_1 c_2) j_2(j_2+1) \right. \\
& \left. + \frac{c_1 + c_2}{2} [j(j+1) - j_m(j_m+1) - i(i+1)] + \frac{c_1 - c_2}{2} \mathbf{R} \cdot (\mathbf{J}_1 - \mathbf{J}_2) \right\}, \quad (1)
\end{aligned}$$

where \mathbf{J}_1 and \mathbf{J}_2 are the grand spins of the P -wave and S -wave kaon, respectively, and $\mathbf{J}_m = \mathbf{J}_1 + \mathbf{J}_2$. The total spin of the system is then given by $\mathbf{J} = \mathbf{J}_{\text{sol}} + \mathbf{J}_m$, where \mathbf{J}_{sol} is the soliton spin. The number and energy of the bound kaons are n_i and ω_i , respectively, and c_i are the hyperfine splitting constants of the bound states. This mass formula consists of three parts. The soliton mass M_{sol} is of $O(N_c)$, where N_c is the number of color, the energy of the bound kaon is of $O(N_c^0)$, and the hyperfine term is of $O(1/N_c)$. Therefore, the mass splitting between the $\Lambda(1405)$ and the $\Lambda(1116)$ mainly comes from the energy difference between the P -wave kaon and the S -wave kaon. In the simple model of Ref. [6], $\omega_2 - \omega_1$ was estimated to be about 200 MeV [8], while its empirical value is about 300 MeV.

In principle, the mass parameters in Eq. (1) should be calculated for a given dynamics of the meson-soliton system. However, this is highly nontrivial because of the complexity of the hadron dynamics. Instead, we fit the parameters to some known hyperon masses and predict the masses of other hyperons. The results obtained in this way are given in Table 2. In this model, the parity of a hyperon changes if the P -wave kaon is replaced by the S -wave kaon. Since the energy difference between the two kaons is about 300 MeV, there are pairs of hyperons of having same spin and the opposite parity having a mass difference of about 300 MeV. Since the mass of the ground state of the $\Xi(1/2^+)$ is 1318 MeV, we can expect to have a $\Xi(1/2^-)$ state at a mass of about 1618 MeV. In our model, there exist two Ξ

Table 2 Mass spectrum of our model. The underlined values are used to determine the mass parameters. The values with the * symbol are obtained by considering the mixing effect. The question mark after the particle name means that the spin-parity quantum numbers are not identified by the Particle Data Group (PDG).

Particle Name	Mass (MeV)	Assigned State
N	<u>939</u>	
Δ	<u>1232</u>	
$\Lambda(\frac{1}{2}^+)$	<u>1116</u>	$\Lambda(1116)$
$\Lambda(\frac{1}{2}^-)$	<u>1405</u>	$\Lambda(1405)$
$\Sigma(\frac{1}{2}^+)$	1164	$\Sigma(1193)$
$\Sigma(\frac{3}{2}^+)$	<u>1385</u>	$\Sigma(1385)$
$\Sigma(\frac{1}{2}^-)$	1475	$\Sigma(1480)?$
$\Sigma(\frac{3}{2}^-)$	1663	$\Sigma(1670)$
$\Xi(\frac{1}{2}^+)$	<u>1318</u>	$\Xi(1318)$
$\Xi(\frac{3}{2}^+)$	1539	$\Xi(1530)$
$\Xi(\frac{1}{2}^-)$	1616(1614*)	$\Xi(1620)?$
$\Xi(\frac{3}{2}^-)$	1658(1660*)	$\Xi(1690)?$
$\Xi(\frac{3}{2}^-)$	<u>1820</u>	$\Xi(1820)$
$\Xi(\frac{1}{2}^+)$	1932	$\Xi(1950)?$
$\Xi(\frac{3}{2}^+)$	<u>2120</u>	$\Xi(2120)?$
$\Omega(\frac{3}{2}^+)$	<u>1694</u>	$\Omega(1672)$
$\Omega(\frac{1}{2}^-)$	1837	
$\Omega(\frac{3}{2}^-)$	1978	
$\Omega(\frac{1}{2}^+)$	2140	
$\Omega(\frac{3}{2}^+)$	2282	$\Omega(2250)?$
$\Omega(\frac{3}{2}^-)$	2604	

states of this mass. This is because the two kaons, one in P -wave and one in S -wave, can make either $j_m = 0$ or $j_m = 1$. When combined with the soliton spin $j_{sol} = 1/2$, these states give two $j = 1/2$ states and one $j = 3/2$ state. This explains naturally the existence of two Ξ baryons with $j^P = 1/2^-$ that have similar masses. In fact, there are candidates for these two Ξ baryons in PDG [22]: the one-star rated $\Xi(1620)$ and the three-star rated $\Xi(1690)$. However, since the observation of the $\Xi(1620)$ at early 1980s [21], there is no other experimental confirmation of this state. Instead, several experiments reported no signal of this state [22]. Therefore, it is strongly required to resolve this issue urgently at current experimental facilities. (See Ref. [23] for a recent experiment for Ξ baryons.)

The above analysis reveals that the $\Xi(1620)$ and the $\Xi(1690)$ are analogue states of the $\Lambda(1405)$. Recently, the BABAR Collaboration claimed that the spin-parity of the $\Xi(1690)$ is $1/2^-$ [24], which supports our prediction. On the other hand, by replacing two P -wave kaons in the $\Xi(1382)$ and in the $\Xi(1530)$, we predict that the $\Xi(1950)$ has $j^P = 1/2^+$ and the $\Xi(2120)$ has $j^P = 3/2^+$. Their spin-parity quantum numbers are not known yet and should be identified by future experiments.

Comparing the predictions presented in Tables 1 and 2 shows that our prediction on the Ω hyperon spectrum is drastically different from the quark model predictions. In quark models, the second lowest Ω hyperon has a mass of around 2 GeV. In our model, the second state has a mass of around 1840 MeV and $j^P = 1/2^-$. Again, we can find that this low mass of the Ω excited state can hardly be explained by quark models. Thus, it is very interesting to see whether such low mass Ω hyperon really exists. Furthermore, most quark models predict that the lowest Ω baryon with $j^P = 1/2^-$ is degenerate or almost degenerate in mass with the lowest Ω baryon with $j^P = 3/2^-$, which is in contradiction to our predictions. These inconsistency with quark model predictions can be tested by future experiments.

If we extend our model to heavy quark baryons [25], we can also find a similar pattern in charm and bottom baryon spectra. Here, one should take into account the center-of-mass problem because of the heavy mass of the charm or bottom meson. In Ref. [26], the binding energies of the soliton-heavy-meson system were calculated in the rest frame of the heavy meson, which shows that the energy difference between the positive parity state and the negative parity state is again close to 300 MeV, which can explain the observed mass difference between the $\Lambda_c(2286)$ of $j^P = 1/2^+$ and the $\Lambda_c(2595)$ of $j^P = 1/2^-$. In quark models, the mass difference between the two states are estimated to be

250 ~ 350 MeV depending on the details of the model on the quark dynamics [11,27]. Therefore, more detailed studies are needed to clarify the structure of the $\Lambda_c(2595)$.

3 Summary

We have analyzed hyperon excited states in the bound state approach in the Skyrme model. This model can explain both the $\Lambda(1116)$ and the $\Lambda(1405)$ on the same footing. We found that the $\Xi(1620)$ and the $\Xi(1690)$ can be regarded as the analogous states of the $\Lambda(1405)$. This model also gives predictions on Ω hyperons that are very different from quark model predictions. However, there is almost no experimental information on the spectrum of Ω baryons. Therefore, detailed studies on the excited states of Ξ and Ω baryons at current experimental facilities are highly required.

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